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# OSCILLATORY PROPERTIES OF A CLASS OF SECOND ORDER QUASI-LINEAR DIFFERENTIAL EQUATIONS

## ABSTRACT
**of thesis for the degree of Doctor of Philosophy (PhD) in the specialty 6D060100 –«Mathematics»**

**Relevance of the topic.** The dissertation is devoted to the problems of oscillation properties of one class of quasilinear difference equations of the second order. In 1836, the French mathematician C.Sturm in his famous 80-page work, was investigating the behavior of order of change of the solution of the second-order linear differential equation's, introduced the concept of oscillations and non-oscillations of the equation, explored the qualitative properties of solutions the second-order linear differential equations, and also proved its well-known theorems of comparison and alternation of zeros as an equation with the property that any solutions have an infinite number of zeros. Since then, research in this area is in full swing and continues to this day. Swanson (C.A. Swanson) summarized the results of the study of linear equations up to the 50s of the last century in his work.

The study of linear differential equations began with the work of A. Elbert and Zh.D.Mirzov, published in the late 70s of the last century. It has been shown that many properties of a linear equation, such as Sturm's comparison theorem, the theorem alternating of zeros, apply to the half-linear differential equation. The Czech mathematician O. Dosly made a great contribution to the study of half-linear differential equations.

The second-order half-linear difference equation was investigated in the 1990s mainly by the Riccati method, and the results up to the year 2000 are given in R.P. Agarwal's book. Since 2001 in the works of P. Rehak and O. Dosly, they brought the theory of the second-order half-linear difference equation to the level of the theory of the second-order half-linear differential equation, established complete analogues of the rotation theorem, Sturm's comparison theorem and generalized the theorem alternating of zeros. In the book of Dosly and P. Rehak 's 2005, they were shows the methods of studying the second-order half-linear differential equation and the second-order half-linear difference equation and presents the results of these equations up to 2005.

General difference equations, regardless of the differential equations, have a great meaning in themselves. Can be applied in mathematical modeling of various phenomena and changes in life and the environment, as well as in the fields of economics, physics, chemistry, biology and discrete dynamic systems. When solving differential equations by numerical method, it is useful to know the oscillations of the solutions of discrete equations. If the solution is non-oscillatory, then can be sure that the numerical calculation works well.

The study of the oscillation properties of the considered equations is based on two equivalent principles given in the rotation theorem.

The first principle is called the Ricatti method, and the second principle is called the variational method. In the Riccati method from the considering equation moves on to the Ricatti equation and examines whether it has a distribution of the solution. However, with respect to the second-order half-linear difference equation, Ricatti's equation is more complex than the second-order half-linear differential equation. Therefore, the analogue of the results obtained by the Ricatti method to the second-order half-linear differential equation have not completely obtained for the second-order half-linear differential equation.

In addition, if the coefficient $\upsilon$ in the second-order half-linear difference equation be a sequence of numbers with different signs, then the study of the equation will be difficult. And the variational method leads to the study of weighted inequalities. Many of the results obtained from the above equations are derived by the Ricatti method.

The variational method is considered in the presence of at least one coefficient of power sequences of the second-order half-linear difference equation. Because in general case the fulfillment of inequality was an unsolved issue. The application of the method of variation in linear equations, differential and difference equations of one and multi variables, and the importance of oscillation and non-oscillation conditions of these equations in the spectral theory of differential operators is well described in the book by N.M. Glasman. However, at least one of the coefficients of the actual differential or difference equations considered in it is a power function or sequence.

If the necessary and sufficient conditions for the fulfillment of the difference inequality according to the second-order half-linear difference equation could be found in terms of coefficients of the second-order half-linear difference equation, then the necessary and sufficient conditions would be found in terms of coefficients of oscillations and non-oscillations of the second-order half-linear difference equation. Thus, the question of solving the difference inequality in accordance with the second-order half-linear difference equation, without limiting the coefficients of the equation, and applying its results to the oscillating properties of the second-order half-linear difference equation remains open.

The dissertation under consideration is devoted to the study of this very topical issue.

In the dissertation, the necessary and sufficient conditions for the fulfillment of the difference inequality with the corresponding to the second-order half-linear difference equation are obtained only in terms of coefficients of the equation, when the case the coefficient $\upsilon$ of the second-order half-linear difference equation is not negative and be a sequence of different sign. On this basis, necessary and sufficient conditions for the oscillating properties of the second-order half-linear difference equation are given. As a consequence of these results, new oscillating properties of the second order linear difference equation emerged. Here, it is assumed that this

coefficient is an arbitrary sequence of real numbers, i.e. members of this sequence can take values of any sign.

The oscillation properties of difference equations are studied by scientists in many countries around the world. For example, scientists from China, America, Japan, Turkey, Saudi Arabia, Egypt, France, Germany, the Czech Republic and Hungary.

The study of the oscillation properties of differential and difference equations in Kazakhstan began with the work of M. Otelbayev and continued by R. Oinarov, L.K. Kusainova, B.S. Koshkarova. On this topic were defended dissertations K.Myrzatayeva's, M. Aldai's, S. Alimagambetov's, S. Kudabayeva's, Kh. Ramazanova's.

**The purpose of the study.** Using the variational method for the second-order half-linear difference equation, in the general case of the coefficients of the equation, find the necessary and sufficient conditions for the fulfillment of the difference inequality with the corresponding to the equation, and on the basis of this, the sufficient and necessary conditions of conjugacy and disconjugate, oscillations and non-oscillations of equation in a given interval give only in terms of coefficients of the equation.

**Object of research.** Two-term second-order half-linear difference equations and difference inequalities.

**Research method.** Using the results of the oscillation properties of the second-order half-linear equation on the basis of the variational method using the conditions for the fulfillment of weighted difference inequalities.

**Scientific novelty of the research.** In addition to the necessary and sufficient conditions for the fulfillment of two-weighted and three-weighted difference inequalities in a set of finite sequences, it is a two-sided estimates of the best constants of inequalities, sufficient and necessary conditions for the oscillation properties of the second coefficient next to the unknown in the second-order two-term half-linear and linear difference equations obtained on the basis of the extended variational principle.

**The main results obtained** for the second-order half-linear and linear difference equations.

*In the presence of any series of non-negative coefficients of the second equation:*

• Extended variational principle;

• Two sided estimates of the best constant of inequality and the necessary and sufficient conditions for the fulfillment of the two-weighted inequality in the set of finite sequences in terms of weighted sequences;

• Necessary and sufficient conditions in terms of coefficients for equations to be conjugacy and disconjugate in a given interval;

• Necessary and sufficient conditions for the oscillations and non-oscillations of the equations in terms of coefficients;

• Necessary and sufficient conditions in terms of coefficients of strong oscillations and strong non-oscillations of equations.

*The second coefficient of the equation is in the case of any series with different signs:*

• Extended variational principle;

• Two-weighted estimates of the best constant of inequality and the necessary and sufficient conditions for the fulfillment of the three-weighted inequality in the set of finite sequences in terms of weighted sequences;

• Necessary and sufficient conditions for the equations to be conjugacy and disconjugate in a given interval in terms of coefficients of oscillations and non-oscillations.

**Theoretical and practical value of the obtained results**. The research has a theoretical nature. The results obtained on weighted difference inequalities can be used in the theory of discrete inequalities and in the theory of discrete operators, in harmonic analysis. The results on the oscillation properties of two-term second-order half-linear partial equations are a contribution to the qualitative theory of difference equations and can be used in the spectral theory of discrete operators and in the numerical solution of differential equations.

**Approbation of the dissertation results.** The main results and scientific conclusions of the dissertation researches were discussed at the scientific seminars "Functional Analysis and Its Applications" organized by the Fundamental Mathematics Department of the L.N. Gumilyov Eurasian National University and also were reported at the traditional international April mathematical conference in honor of the Day of Science Workers of the Republic of Kazakhstan and Workshop "Problems of modeling processes in electrical contacts", Almaty, April 3-5, 2019; the international scientific conference "Theoretical and applied issues of mathematics, mechanics and computer science", Karaganda, June 12-13, 2019; the international conference "Actual problems of analysis, differential equations and algebra"(EMJ-2019), Nur-Sultan, October 16-19, 2019; the international Uzbek-Russian scientific conference "Non-classical equations of mathematical physics and their applications", Tashkent, October 24-26, 2019.

In addition, during the scientific internship between September 24 and December 27, 2018, Professor of "Mathematics and Statistics", Doctor of Physical and Mathematical Sciences, Honored Worker of Science of the Russian Federation in the field of science, reports were made at scientific seminars "Spectral theory of differential operators" led by the laureate of the State Prize of the Republic of Russia Sultanaev Yaudat Talgatovich (Russian Federation, the Republic of Bashkortostan, Ufa, M. Bashkortostan State Pedagogical University named after Akmulla, Faculty of Physics and Mathematics).

**Publications.** The main results of the dissertation research were published in 11 papers, including 3 articles in the publications recommended by the Committee for Control in Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan, 2 article in a journal from the Scopus database with a CiteScore percentile of at least 25 and 6 theses in the materials of international scientific conferences, including 1 work in the collection of abstracts of articles of conferences abroad.

**The structure of the dissertation**. The dissertation consists of an introduction, two chapters, a conclusion and a list of references. Each section is divided into subsections.